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The influence of cooling rate and $SiO₂$ additions on the grain boundary structure of Mn-doped PTC thermistors

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Abstract

BaTiO₃ based positive temperature coefficient of resistance (PTC) thermistors were prepared with 0, 1.0, 2.0 and 3.0 at.% SiO₂ additions. The effects of these SiO2 additions and cooling rate variations on microstructural development and bulk electrical performance are discussed in the context of the double Schottky barrier (DSB) model. An increase in $SiO₂$ content increased the proportion of triple junctions containing BaTiSi₂O₈, but reduced the density slightly, without affecting the grain size. The bulk charge carrier density was unaffected either by SiO₂ level or changes to the cooling rate. Adding SiO₂ or increasing the post sintering cooling rate decreased ρ_{25} and ρ_{max} , and increased $T\rho_{\text{max}}$. These effects were attributed to a reduction in activated surface state density, while maintaining a fixed acceptor energy depth.

By assuming a single acceptor state energy level, satisfactory agreement between theory and experimental PTC behaviour was only obtained at *T*ρmax. Agreement was extended over a wider range of temperatures when the acceptor states were considered to be spread over a broader energy interval.

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1. Introduction

Positive temperature coefficient of resistance (PTC) thermistors, based on donor and acceptor co-doped $BaTiO₃$, exhibit an anomalous rise in grain boundary resistivity over a small temperature interval just above the ferro- to para-electric transition temperature, $T_{\rm C}$. A mechanism for this process was first pro-posed by Heywang^{[1](#page-9-0)} and subsequently developed by Jonker.^{[2,3](#page-9-0)} In this model the sudden increase in grain boundary resistivity above $T_{\rm C}$ is associated with the formation of a Schottky-type potential barrier within a permittivity-dependant depletionlayer. The barrier is due to filled acceptor traps, formed within a 2D layer of segregated acceptor ions,^{[4](#page-9-0)} or adsorbed oxygen^{[5](#page-9-0)} at the grain boundaries, and disappears below T_C due to charge compensation by spontaneous polarization. An alternative model for the formation of a permittivity-dependant interfacial potential barrier where cation vacancies, mainly Ba-vacancies, act as acceptors was proposed by Daniels and Wernicke.^{[6](#page-9-0)} In their model, samples prepared with an appropriate donor concentration and sintered in an atmosphere containing a suitable oxygen partial pressure form a Ba-vacancy rich insulating layer at the grain boundaries, resulting in an n-i-n type interfacial structure. The resistivity of the depletion-layer, ρ_L , is approximately given by $1,7$:

$$
\rho_{\rm L} = R_0 \, \exp\left\{\frac{e\varphi_0(T)}{kT}\right\} \tag{1}
$$

where, R_0 = constant, e = electronic charge, $\varphi_0(T)$ = height of the potential barrier at temperature *T*, and *k* = Boltzmann constant. $\varphi_0(T)$ can be obtained from the solution of the 1D Poisson equation:^{[8](#page-9-0)}

$$
\varphi_0(T) = \frac{eN_S(T)^2}{8\varepsilon_0\varepsilon_L'(T)N_d} \tag{2}
$$

where, $N_S(T)$ = density of occupied acceptor states at temperature *T*, ε_0 = permittivity of free space, N_d = charge carrier (effective donor) concentration. ε'_{L} is the relative permittivity of the material within the grain boundary layer, which can be

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described through the Curie–Weiss law:

$$
\varepsilon_{\rm L}^/ = \frac{C}{T - T_{\rm C}}\tag{3}
$$

where $C =$ Curie constant.

Addition of 3-d elements, especially Mn, is believed to increase the magnitude of the PTC effect through the formation of deep grain boundary acceptor states.[9](#page-9-0) Experimental evidence for segregation of Cu, Fe and Mn to secondary grain boundary phases led early workers to postulate this as an explanation for the observation that only small additions are necessary to achieve a large increase in the magnitude of the overall resis-tance increase during the PTC transition.^{[1,10](#page-9-0)} However, it has also been noted that many transition elements have high solubility limits in BaTiO₃, and that only Cu and V are likely to segregate to secondary phases, while others distribute homogeneously throughout the structure.^{[9](#page-9-0)}

The n-i-n layer model^{[6](#page-9-0)} allows for a homogeneous distribution of acceptor ions throughout the lattice, and assumes the majority of donor states are compensated by Ba-vacancies, reducing the effective donor concentration to a level similar to that of the acceptors. In this model the addition of acceptor levels with higher energies than that of the Ba-vacancy increases the thickness of the insulating layer in the n-i-n structure, increasing the magnitude of the resistivity change during the PTC transition.

In thermistors co-doped with La and Mn, charge neutrality arguments have been used to suggest that diffusion of atmospheric oxygen during cooling generates electron traps, leading to oxidation of Mn-ions at the grain surfaces. $11,12$

 $SiO₂$ and TiO₂ are commonly added as sintering aids.^{[13,14](#page-9-0)} $SiO₂$ and Mn additions are also reported to assist homogenization of the microstructure during sintering, improving the performance of PTC thermistors under transient loads.^{[15](#page-9-0)} In terms of electrical properties, the variation of permittivity with temperature is significantly affected by $SiO₂$ additions: with increasing $SiO₂$ the permittivity increases below T_C , although the permittivity close to the temperature of maximum resistivity, $T_{\rho_{\text{max}}}$, remains almost unchanged.^{[16](#page-9-0)} It has been reported that increasing the $SiO₂$ content in La-doped BaTiO₃ initially causes an increase in room temperature resistivity, followed by a sudden decrease. The initial rise was attributed to homogenization of the microstructure through the formation of a liquid phase, and the subsequent decrease to fresnoite formation, which reduced the surface Ba-vacancy concentration by reducing the contact area between intergranular $Ba₆Ti₁₇O₄₀$, and $BaTiO₃$ grains.[17](#page-9-0)

In the present study we have systematically analysed resistance–temperature $(\rho(T))$ curves from series of PTC thermistors, formed as a function of cooling rate and $SiO₂$ content, in the framework of the double Schottky barrier (DSB) model, in order to establish the effect of variations in these process parameters on the electrical structure of the grain boundaries. We have done this through a combination of microstructural studies and extension of an analysis describing the effect of variations in surface state density, N_{SO} , and acceptor level energy, E_{S} , on $\rho(T)$ ^{[18](#page-9-0)} to include subsequent work, which generalised N_{SO} and E_S in $\rho_{\text{max}} - T_{\text{max}}$ space^{[9](#page-9-0)} and trap energy distributions.^{[19](#page-10-0)} These are compared with values obtained through Arrhenius analy-sis of the same data.^{[7](#page-9-0)} We will demonstrate that SiO_2 additions systematically alter the low field $\rho(T)$ characteristics of PTC thermistors above $T_{\rm C}$ in a manner directly analogous to changes in cooling rate.

Fig. 1. Backscattered electron images of samples with (a) 0 at.% SiO_2 , (b) 1.0 at.% SiO_2 , (c) 2.0 at.% SiO_2 and (d) 3.0 at.% SiO_2 . Scale bar = 10 μ m.

Fig. 2. Graphs showing the variation in grain size with $SiO₂$ content for each cooling rate.

2. Experimental

The thermistor samples used in this study were prepared from (Ba,Ca)TiO₃, doped with a rare earth donor and 0.1 at.% Mn acceptor. Samples were prepared with four different $SiO₂$ levels by including 0, 1.0, 2.0 and 3.0 at.% SiO₂. Each batch was placed in a polyethylene container with Mg-PSZ grinding media, using a powder to media ratio of 1:1. Isopropyl alcohol $[(CH₃)₂CHOH]$ was added at an appropriate level to form a thin slurry, and the mixtures were milled in a vibratory mill for 18 h, followed by drying for 6 h at 85° C in air. Disc compacts, 10 mm diameter and 2.2 mm thick, were pressed at a pressure of 8 kg mm[−]2. Green pellets were sintered in air in a muffle furnace at a peak temperature of 1330 ◦C for 1 h with cooling rates of 150, 300 and $600 \degree C \text{ h}^{-1}$, producing sintered pellets 8 mm in diameter and 2 mm in thickness.

The sintered pellets were cleaned thoroughly in acetone, and In–Ga (2:1) eutectic alloy electrodes were applied to form ohmic contacts. Low field resistivity measurements were carried out in the temperature range $30-300$ °C, at 10 °C intervals, by employing a two probe DC technique using a KEITHLEY 487 Picoammeter with a built-in voltage source. Samples were held at temperature for 30 min before each measurement to ensure thermal equilibration. Resistivity was calculated at each temperature using the average of 10 measurements.

Impedance measurements were carried out on samples approximately 1 mm thick and 8 mm diameter using a HP3192A

Fig. 3. Graphs showing (a) relative density as a function of $SiO₂$ content, compensated for $SiO₂$ content, and (b) open porosity as a function of $SiO₂$ content.

impedance analyser, operating over a frequency range of 5 Hz to 13 MHz. Data were collected in the temperature range 30–450 \degree C, at 10 \degree C intervals. Samples were held at temperature for 30 min prior to each measurement in order to ensure thermal equilibration. The real and imaginary components of impedance were obtained as a function of frequency and temperature, and were corrected for the effects of the sample holder and leads.

Microstructural analysis was carried out on polished crosssections of the specimens using backscattered electron (BS) imaging in a Phillips XL30 FEGSEM operating at 10 kV. The average grain size was measured using the linear intercept method with a *k* value of 1.6. Sample density and porosity was measured using the Archimedes' method and compared with the theoretical density of $(Ba,Ca)TiO₃$ after compensating for $SiO₂$ addition. X-ray diffraction was carried out using a Phillips

Fig. 4. XRD of samples as a function of $SiO₂$ content. Peaks labelled 'A' and 'B' are attributed to (Ca,Ba)TiO₃ and BaTiSi₂O₈, respectively.

X'pert APD Diffractometer, operating at 60 kV and 4 mA, fitted with PC-APD diffraction software. The samples were scanned over a 2 θ range of 5–95 \degree , using a 0.05 \degree step scan.

3. Results and discussion

3.1. Microstructure

The microstructures of the samples cooled at 300 °C h⁻¹ are shown in [Fig. 1a](#page-1-0)–d, for each $SiO₂$ level. A $SiO₂$ -rich second phase is predominantly observed at triple junctions, and is visible in all samples containing at least $1 \text{ at.} %$ SiO₂, suggesting the presence of a transient liquid phase during sintering. As the $SiO₂$ is increased, the proportion of triple junctions appearing to contain a secondary silicate phase increases. Additionally, the predominant grain shape changes from rounded to ${100}$ habit, indicating that the relative energies of different $BaTiO₃$ surfaces are modified by the presence of excess $SiO₂$, although the mean grain size remains largely unchanged ([Fig. 2\).](#page-2-0) Similar microstructures were observed in samples cooled at 150 and $600 °C h^{-1}$.

The densities of the samples, when corrected for $SiO₂$ content, all lie in the range 96.5–98.5%, and decrease with increasing $SiO₂$ ([Fig. 3a](#page-2-0)). Due to the lower density of silicate compared with $(Ba, Ca)TiO₃$, the titanate grain structure becomes progressively more 'open' as SiO₂ increases, i.e. for the samples with 0 at.% $SiO₂$ addition the titanate grains are more closely packed with a few pores trapped at the grain boundaries or triple points, but with increasing $SiO₂$ the apparent grain-to-grain contact area decreases. The proportion of open porosity was calculated as a function of $SiO₂$ addition, using the Archimedes' method, and was found to decrease smoothly from $0.70 \pm 0.05\%$ to $0.35 \pm 0.03\%$ as the SiO₂ level increased from 0 to 3 at.% ([Fig. 3b](#page-2-0)).

XRD analysis [\(Fig. 4\)](#page-2-0) identified the second phase as fresnoite, $Ba₂TiSi₂O₈$. The volume of fresnoite in the microstructure increases with $SiO₂$ addition, and appeared slightly more abundant in the more rapidly cooled samples.

In Ti-rich BaTiO₃ the formation of a Ba₆Ti₁₇O₄₀ second phase is widely reported, formed by the reaction of BaTiO₃ with excess $TiO₂$. ^{[17](#page-9-0)} This phase is also reported to form in samples containing $SiO₂$ additions, along with a slightly Ba-rich fresnoite, according to the following reaction 17 :

$$
28\text{BaTiO}_3 + 22\text{SiO}_2 \Leftrightarrow 11\text{Ba}_2\text{TiSi}_2\text{O}_8 + \text{Ba}_6\text{Ti}_{17}\text{O}_{40} \tag{4}
$$

However, although fresnoite was observed in our samples, no $Ba₆Ti₁₇O₄₀$ was detected.

3.2. Electrical properties

Fig. 5a shows typical $\rho(T)$ curves for thermistors cooled at 150° C h⁻¹, as a function of SiO₂ content, while Fig. 5b shows $\rho(T)$ curves for thermistors containing 1% SiO₂, as a function of cooling rate. Similar relationships between the $\rho(T)$ curves were seen for other cooling rates and $SiO₂$ additions. Increasing the $SiO₂$ content or the sample cooling rate decreased the room

Fig. 5. $\rho(T)$ curves as a function of (a) SiO₂ content for samples cooled at 150° C h⁻¹ and (b) cooling rate for samples containing 1 at.% SiO₂.

temperature resistivity, ρ_{25} , and the maximum resistivity, ρ_{max} , while reducing the gradient of the PTC portion of the $\rho(T)$ curve, and increasing the temperature of maximum resistivity, $T_{\rho_{\text{max}}}$.

[Fig. 6a](#page-4-0) and b shows the variations in ρ_{RT} and ρ_{max} , respectively, as a function of cooling rate, and $SiO₂$ level, for all samples. The rate of variation increases as the $SiO₂$ level is reduced. Additionally, the rate of increase of ρ_{25} and ρ_{max} with decreasing cooling rate decreases as the amount of $SiO₂$ added to the starting powder is increased.

It is well known that the addition of acceptor impurities, such as manganese, increases the magnitude of the resistivity jump $(\rho_{\text{max}}/\rho_{25})$, ^{[20,21](#page-10-0)} through the compensation of Mn'_{Ti} substitutional defects by oxygen vacancies, $V_0^{\bullet\bullet}$, in order to maintain

Fig. 6. Variation in (a) ρ_{25} and (b) ρ_{max} with cooling rate for each SiO₂ content.

overall charge neutrality, according to:

$$
MnO \frac{mO_2}{mH} Mn''_{\text{Ti}} + O_O^{\text{x}} + V_O^{\bullet \bullet}
$$
 (5)

During reoxidation on cooling, oxygen vacancies are lost and the Mn-ions at the grain boundaries form acceptor states. 22 22 22 Thus, during reoxidation $N_S(T)$ increases, leading to an associated increase in $\varphi_0(T)$, according to Eq. [\(2\).](#page-0-0) Extending the time available for reoxidation, e.g. by decreasing cooling rate, allows for a greater increase in $N_S(T)$, giving rise to a bigger PTC resistivity jump. Changes in PTC curve position and shape with cooling rate are therefore explained simply in terms of changes in time available, and hence the extent of reoxidation during cooling.

A variation in curve position with increasing $SiO₂$ content, similar to that observed for an increase in cooling rate, occurred within our samples. This can be understood in terms of microstructural changes since with increasing addition of $SiO₂$ the proportion of triple junctions occupied by fresnoite increases and the amount of open porosity progressively decreases. Both these observations imply a reduction in available pathways for oxygen migration within the thermistor, and hence reoxidation during cooling. This is illustrated schematically in Fig. 7a–d. There is also the possibility that the increased volume of secondary phase in the high $SiO₂$ samples will dissolve more Mn during sintering, reducing the concentration of Mn available for

Fig. 7. (a–d) Schematic diagram showing a progressive increase in the proportion of blocked grain boundaries (crosses) with increasing triple point second phase (spots).

acceptor state formation. However, we were unable to confirm this through TEM analysis due to the low concentration of Mn present. All of these processes would reduce $N_S(T)$, reducing both the residual barrier below T_{C} and the interface potential barrier above $T_{\rm C}$, leading to reductions in resistivity in both regions. Previous workers^{[17](#page-9-0)} have discussed a similar variation in ρ_{25} with SiO₂ addition in terms of increased fresnoite content, which prevents the formation of Ba-vacancies by inhibiting the reaction between BaO and $Ba_6Ti_{17}O_{40}$ to form BaTiO₃. Since no $Ba₆Ti₁₇O₄₀$ was observed in our samples, the mechanism they describe is unlikely in our case.

Thus, for a given composition, the position and shape of the PTC curve simply reflects the extent of grain boundary oxidation in the material, and is influenced both by cooling rate and $SiO₂$

Fig. 8. Arrhenius plots for each cooling rate in samples containing (a) 1.0 at.% SiO₂ and (b) 2.0 at.% SiO₂.

Fig. 9. Effect of cooling rate and $SiO₂$ content on the variation in surface state density, calculated from the Arrhenius plots.

level. We also note that the effect of cooling rate on ρ_{25} and ρ_{max} is observed to be more significant at low $SiO₂$ levels [\(Fig. 6\),](#page-4-0) possibly reflecting the importance of open oxygen migration pathways on the reoxidation kinetics.

According to the DSB model^{[1](#page-9-0)} the initial gradient of the $\rho(T)$ curve, $d\rho/d(1 - T_C/T)$, just above T_C should remain constant provided the occupied acceptor state density remains unaffected by the relative shift of surface state energy level with respect to the Fermi level inside the forbidden zone and the relative permittivity varies according to the Curie–Weiss law. On this basis, combining Eqs. (1) – (3) gives:

$$
\rho_{\rm L} = R_0 \exp \frac{e^2 (N_{\rm S}(T))^2 (T - T_{\rm C})}{8 \varepsilon_0 C k N_{\rm d} T} \tag{6}
$$

At temperatures near T_C all the acceptor states are fully occupied, 7,19 7,19 7,19 and so, $N_S(T)$ is equal to the total activated surface state density N_{SO} . As a result ρ_L increases rapidly with

Table 1 Values of surface state density and energy, extracted from $\rho_{\text{max}}-T_{\text{max}}$ plots and Arrhenius plots for a single value of E_S

| Cooling rate ($^{\circ}$ Ch ⁻¹) | $SiO2$ content (at.%) | Single acceptor energy | | $N_{\rm SO}$ (cm ⁻²) from Arrhenius graphs | |
|--|-----------------------|----------------------------------|------------|--|--|
| | | $N_{\rm SO}$ (cm ⁻²) | E_S (eV) | | |
| 150 | $\overline{0}$ | 1.09×10^{14} | 1.51 | 1.08×10^{14} | |
| | 1.0 | 0.97×10^{14} | 1.54 | 0.97×10^{14} | |
| | 2.0 | 0.92×10^{14} | 1.55 | 0.86×10^{14} | |
| | 3.0 | 0.86×10^{14} | 1.55 | 0.74×10^{14} | |
| 300 | θ | 0.95×10^{14} | 1.49 | 0.89×10^{14} | |
| | 1.0 | 0.90×10^{14} | 1.51 | 0.83×10^{14} | |
| | 2.0 | 0.80×10^{14} | 1.51 | 0.74×10^{14} | |
| | 3.0 | 0.77×10^{14} | 1.53 | 0.71×10^{14} | |
| 600 | θ | 0.82×10^{14} | 1.58 | 0.72×10^{14} | |
| | 1.0 | 0.77×10^{14} | 1.55 | 0.67×10^{14} | |
| | 2.0 | 0.76×10^{14} | 1.57 | 0.67×10^{14} | |
| | 3.0 | 0.71×10^{14} | 1.58 | 0.64×10^{14} | |

Table 2

Values of surface state density and energy, extracted from $\rho_{\text{max}}-T_{\text{max}}$ plots for different values of Δ

| Cooling rate $({}^{\circ}Ch^{-1})$ | $SiO2$ content (at.%) | Acceptor energy spread (eV) | | | | | | |
|------------------------------------|-----------------------|----------------------------------|------------|----------------------------------|------------|----------------------------------|------------|--|
| | | 0.25 | | 0.5 | | 0.75 | | |
| | | $N_{\rm SO}$ (cm ⁻²) | E_S (eV) | $N_{\rm SO}$ (cm ⁻²) | E_S (eV) | $N_{\rm SO}$ (cm ⁻²) | E_S (eV) | |
| 150 | $\overline{0}$ | 1.11×10^{14} | 1.55 | 1.16×10^{14} | 1.60 | 1.37×10^{14} | 1.58 | |
| | 1.0 | 0.98×10^{14} | 1.57 | 1.01×10^{14} | 1.65 | 1.08×10^{14} | 1.69 | |
| | 2.0 | 0.92×10^{14} | 1.58 | 0.94×10^{14} | 1.65 | 0.98×10^{14} | 1.71 | |
| | 3.0 | 0.87×10^{14} | 1.59 | 0.88×10^{14} | 1.66 | 0.91×10^{14} | 1.73 | |
| 300 | $\overline{0}$ | 0.96×10^{14} | 1.52 | 0.98×10^{14} | 1.60 | 1.06×10^{14} | 1.63 | |
| | 1.0 | 0.91×10^{14} | 1.55 | 0.93×10^{14} | 1.62 | 0.97×10^{14} | 1.68 | |
| | 2.0 | 0.81×10^{14} | 1.55 | 0.82×10^{14} | 1.62 | 0.84×10^{14} | 1.70 | |
| | 3.0 | 0.77×10^{14} | 1.57 | 0.78×10^{14} | 1.65 | 0.80×10^{14} | 1.73 | |
| 600 | $\overline{0}$ | 0.82×10^{14} | 1.58 | 0.83×10^{14} | 1.65 | 0.85×10^{14} | 1.73 | |
| | 1.0 | 0.76×10^{14} | 1.55 | 0.77×10^{14} | 1.62 | 0.79×10^{14} | 1.70 | |
| | 2.0 | 0.76×10^{14} | 1.57 | 0.77×10^{14} | 1.65 | 0.78×10^{14} | 1.73 | |
| | 3.0 | 0.71×10^{14} | 1.58 | 0.72×10^{14} | 1.66 | 0.73×10^{14} | 1.74 | |

Fig. 10. A schematic band diagram for the interface potential barrier with acceptor levels, (a) concentrated in a single energy state and (b) continuously distributed within an energy range of Δ .

temperature above $T_{\rm C}$. Eq. [\(6\)](#page-5-0) can be written in Arrhenius form:

$$
\ln \rho_{\rm L} = A \left(\frac{T - T_{\rm C}}{T} \right) + \ln R_0 \tag{7}
$$

where *A* is a constant, representing the gradient of the Arrhenius plot. Hence:

$$
N_{\rm SO} = \sqrt{\frac{8A k \varepsilon_0 C N_{\rm d}}{e^2}}\tag{8}
$$

Two typical examples of Arrhenius plots are shown in [Fig. 8](#page-4-0) for which a $T_{\rm C}$ of 130 °C was used. There is some deviation of the experimental points from linearity at higher temperatures, which is due to the effect of depopulation of surface states as they approach the Fermi level. Values of $N_{\rm SO}$, calculated from the initial gradients using Eq. (8) , with respect to $SiO₂$ and cooling rate, are presented in the final column of [Table 1](#page-5-0) and in [Fig. 9.](#page-5-0) In these calculations, a value of N_d of 1.41×10^{18} cm⁻³ was used, being obtained from the highest frequency intercept in a complex impedance plot in the usual way. It can be seen from these curves that the surface state density gradually increases with a decrease in either cooling rate or $SiO₂$ content, which is consistent with the explanation for variation in curve position given earlier.

3.3. Theoretical analysis

For a polycrystalline thermistor with *z* grains cm^{-1} between the electrodes, the effective resistivity including the interface

Fig. 11. $\rho_{\text{max}} - T_{\text{max}}$ plots for a single energy state.

potential barrier is given by²:

$$
\rho(T) = \left(\frac{1}{N_{\rm d}e\mu}\right) \left[1 + \left\{\frac{zb(T)kT}{e\varphi(T)}\right\} \exp\left\{\frac{e\varphi(T)}{kT}\right\}\right] \tag{9}
$$

where, $b(T)$ is the barrier layer width, given by $\{N_S(T)\}/(2N_d)$, and μ is the electron mobility, here taken a 0.5 cm² Vs⁻¹. The value for *z* was based on an average sample grain size of $6.4 \mu m$.

Fig. 12. $\rho_{\text{max}}-T_{\text{max}}$ plots for (a) $\Delta = 0.25$ eV, (b) $\Delta = 0.5$ eV and (c) $\Delta = 0.75$ eV.

The value of $N_S(T)$ can be represented in the form of a Fermi distribution function^{1,2}:

$$
N_{\rm S}(T) = \frac{N_{\rm SO}}{1 + \exp\{E_{\rm F} + e\varphi(T) - E_{\rm S}/kT\}}\tag{10}
$$

where, E_S = energy of grain boundary acceptor levels below the conduction band edge, assuming all acceptors are concentrated in a single energy state within the forbidden zone ([Fig. 10a\)](#page-6-0). E_F is the Fermi energy, which is given by;

$$
E_{\rm F} = kT \ln \left(\frac{N_0}{N_{\rm d}}\right) \tag{11}
$$

where, N_0 = density of states, taken as 1.56×10^{22} cm⁻³, corresponding to the density of Ti ions. $23-25$.

In order to model the $\rho(T)$ characteristics, the total surface state density, N_{SO} , and the surface state energy, E_S , were established for each sample by solving Eqs. [\(2\) and \(10\)](#page-0-0) for a constant value of effective donor concentration using a Newton–Raphson technique and assuming Curie–Weiss behaviour. The calculations were repeated for several values of surface state density and acceptor depth, and then over a range of temperatures above *T*_C to obtain values of effective, or occupied, trap densities and barrier heights as a function of temperature. Values of ρ_{max} and *T*max were extracted using Eq. [\(9\)](#page-6-0) for a constant trap density and trap energy levels in the range 1.2–1.8 eV. Then a $\rho_{\text{max}}-T_{\text{max}}$ plot was constructed as a function of effective surface state density and surface state energy, in the manner described by Jonker.^{[18](#page-9-0)} Our experimental ρ_{max} and T_{max} data were superimposed on this plot to extract the values of N_{SO} and E_{S} as a function of $SiO₂$ content and cooling rate [\(Fig. 11\).](#page-6-0) These values are listed in [Table 1. I](#page-5-0)t can be seen that N_{SO} decreases with an increase in either the $SiO₂$ content or cooling rate. Values of N_{SO} obtained from the $\rho_{\text{max}}-T_{\text{max}}$ plots are close to those obtained from the Arrhenius plots, also listed in [Table 1.](#page-5-0) Our values of E_S all

Fig. 13. Calculated values of resistivity and $N_S(T)$ as a function of temperature for (a) four different $SiO₂$ contents an (b) three different cooling rates.

Fig. 14. Calculated values of PTC shape factor $(e\varphi_0/kT)$ and interface potential barrier height ($e\varphi_0$) to illustrate Fermi pinning as a function of (a) SiO_2 content and (b) cooling rate.

fall within the range 1.51–1.55 eV, which is slightly higher than other published values for Mn-doped BaTiO₃ PTC thermistors $(1.40-1.47 \text{ eV})$.^{[22](#page-10-0)} We believe this discrepancy is consistent with differences between the two sets of samples: our Mn-doped (Ba,Ca)TiO₃ samples exhibit a higher ρ_{max} (10⁷–10⁹ Ω cm) than those in the previously published study $(10^5-10^8 \Omega \text{ cm})$, although values of T_{max} are similar (220–280 °C).

Values of total surface state density and acceptor energy depth were also calculated for the case where there is a continuous distribution of acceptor states, spread over a narrow energy range, Δ , centred on E_S , as shown schematically in [Fig. 10b,](#page-6-0) using the analysis proposed by Ihring and Puschert.[19](#page-10-0) A constant number of states per unit area and unit energy n_{so} is also assumed for simplicity. Using these assumptions, Eq. (10) is modified to¹⁹:

to extract corresponding values of $N_{\rm SO}$ and $E_{\rm S}$ for our samples as a function of Δ , cooling rate and SiO₂ content [\(Table 2\).](#page-5-0) Increases in $SiO₂$ or cooling rate both reduce N_{SO} irrespective of Δ. The gradual shift in surface state energy levels towards higher values with increasing Δ may partly be due to a reduction in the energy difference between the Fermi level and the upper edge of the distributed surface states band, which will automatically increase E_S , provided the temperature difference between T_C and $T\rho_{\text{max}}$ is fixed.

[Fig. 13a](#page-7-0) and b shows the calculated values of resistivity and $N_S(T)$ as a function of $SiO₂$ content and cooling rate. [Fig. 14a](#page-7-0) and b shows the relation between E_F , the interface conduction band edge energy, GBE_C , and the acceptor energy level in the forbidden zone with respect to temperature. The resistivity reaches a

$$
N_{\rm S}(T) = n_{\rm SO}kT \left[\frac{\Delta}{kT} - \ln \frac{1 + \exp\{e\varphi(T) + E_{\rm F} - E_{\rm S} + (1/2)\Delta/kT\}}{1 + \exp\{e\varphi(T) + E_{\rm F} - E_{\rm S} - (1/2)\Delta/kT\}} \right]
$$

 $\rho_{\text{max}}-T_{\text{max}}$ plots were constructed in a similar fashion for Δ values of 0.25, 0.5 and 0.75 eV as a function of n_{so} and E_S and are presented in [Fig. 12a](#page-6-0)–c. The experimental ρ_{max} and T_{max} data were superimposed on each of the $\rho_{\text{max}}-T_{\text{max}}$ plots in order

maximum value at a temperature near to the onset of depopulation of surface states. The temperature at which depopulation starts increases with both $SiO₂$ content and cooling rate. By

Fig. 15. Comparison between the theoretical and experimental ρ –*T* characteristics for four different values of Δ for samples with (a) 0 at.% SiO₂, 150 °C h⁻¹, (b) 1.0 at.% SiO₂, 150 °C h⁻¹, (c) 2.0 at.% SiO₂, 150 °C h⁻¹, (d) 3.0 at.% SiO₂, 150 °C h⁻¹ and (e) 0 at.% SiO₂, 300 °C h⁻¹.

(12)

comparing [Figs. 13a and 14a](#page-7-0) it can be seen that surface state depopulation begins when the surface state energies approach the Fermi level due to the rise in interface potential barrier height with increase in temperature. Since the interfacial potential barrier increases more slowly in the samples with the higher $SiO₂$ content, Fermi pinning and consequently the depopulation of surface states is delayed, resulting in a higher value of $T_{\rho_{\text{max}}}$. Comparison of [Figs. 13b and 14b](#page-7-0) shows that an increase in cooling rate also shifts the onset of Fermi pinning, and hence the temperature corresponding to onset of surface state depopulation, to higher temperatures. This is achieved through a reduction in the rate of rise of the interface potential barrier with respect to temperature $(d\varphi_0/dT)$, in a similar manner to that for increasing SiO₂ content. The reduction in $d\varphi_0/dT$ is therefore solely due to a reduction in *N*_{SO}.

 $\rho(T)$ characteristics were calculated using the parameters tab-ulated in [Tables 1 and 2,](#page-5-0) for a range of Δ , and compared with the experimental $\rho(T)$ curves from our samples ([Fig. 15a](#page-8-0)–e). As Δ increases the discrepancy between the calculated and the experimental curves gradually decreases for all $SiO₂$ levels and cooling rates, especially within the high temperature NTC region and just below $T_{\rho_{\text{max}}}$. For low values of Δ the calculated resistivity peak is quite sharp, whereas for Δ values between 0.5 and 0.75 eV dρ/d*T* the resistivity change with respect to temperature around $T_{\rho_{\text{max}}}$ broadens with a larger initial PTC slope and hence a larger value of N_{SO} and there is better agreement between theory and experiment around peak resistivity and within the NTC region. The residual discrepancy is attributed to an overestimate of the permittivity of the grain boundary layer through the use of the Curie–Weiss law, and is supported by the observation that lower permittivity values have been calculated from single grain boundary capacitance measurements, $2⁶$ together with evidence that grain boundary defects and internal stresses can also reduce the actual permittivity above T_{C} .^{[19](#page-10-0)} An additional factor affecting the fit may be the simplistic assumption of a continuous distribution of acceptor states and a constant density of states per unit energy $n_{\rm so}$, given the coexistence of Ba-vacancies, impurities and dopants.[19](#page-10-0)

4. Summary

 Co -doped $(Ba,Ca)TiO₃$ based PTC thermistor samples were prepared with additions of 0, 1.0, 2.0 and 3.0 at.% $SiO₂$ and with cooling rates of 150, 300 and $600^{\circ} \text{C h}^{-1}$. An increase in $SiO₂$ content was found to decrease the sample density, reduce grain-to-grain contact area, decrease the volume fraction of open porosity and increase the proportion of silicate second phase present at triple junctions.

 $SiO₂$ has long been added to BaTiO₃ as a sintering aid, but has not been reported to affect significantly the electrical properties of PTC thermistors. In this systematic study we have observed that additions of $SiO₂$ and increases in sample cooling rate both modify the electrical characteristics by decreasing ρ_{25} , ρ_{max} and $d\rho/dT$, whilst displacing $T\rho_{\text{max}}$ towards higher temperatures. Variations in $SiO₂$ content and cooling rate were not found to affect the bulk resistivity.

The variation in low field $\rho(T)$ behaviour was analysed as functions of $SiO₂$ content and cooling rate over the full range of the PTC transition using two different models, one based on an Arrhenius analysis and the other on direct modelling of ρ –*T* behaviour above T_C . It was observed for the first time that $SiO₂$ additions affect the interfacial electrical parameters, N_{SO} and *E*S, in a way that is directly analogous to changes in cooling rate, both decreasing N_{SO} , while E_S remained unchanged within the range 1.51–1.55 eV. The consistency of data obtained using these two distinct models, both in terms of absolute values and variation with changes in composition and/or processing adds weight to this interpretation.

These systematic changes in electrical behaviour with $SiO₂$ addition were attributed to a reduction in activated acceptor state density in the grain boundary regions due to incomplete reoxidation caused by a progressive blocking of triple junctions and a reduction in open porosity, together with possible dissolution of Mn into an increasing volume of silicate phase during sintering.

The agreement between theoretical and experimental $\rho(T)$ characteristics was found to be poor when the surface state energy levels were confined to a single energy level. Agreement gradually improved, especially around $T\rho_{\text{max}}$ and in the high temperature NTC region, when the surface states were considered to be distributed over a broader energy range $(\Delta = 0.5 - 0.75 \text{ eV})$. A discrepancy between the calculated and experimental ρ –*T* curves between T_C and $T_{\rho_{\text{max}}}$ may be due to the grain boundary layer having a lower permittivity than predicted by the Curie–Weiss law and complexity in the surface state distribution.

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References

- 1. Heywang, W., *J. Amer. Ceram. Soc.*, 1964, **47**, 484.
- 2. Jonker, G. H., *Solid State Electron.*, 1964, **7**, 895.
- 3. Jonker, G. H., Grain boundary phenomena in electronic ceramics. *Am. Ceram. Soc.*, 1981, 155.
- 4. Heywang, W., *J. Mater. Sci.*, 1971, **6**, 1214.
- 5. Kuwabara, M., *Solid State Electron.*, 1984, **27**, 929.
- 6. Daniels, J. and Wernicke, R., *Philips Res. Repts.*, 1976, **31**, 544.
- 7. IIIingsworth, J., Al-Allak, H. M. and Brinkman, A. W., *J. Phys. D: Appl. Phys.*, 1990, **23**, 971.
- 8. Hozer L, Semiconductor Ceramics: Grain Boundary Effects, Ellis Horwood, Polish Scientific Publishers PWN, 1994.
- 9. Ihring, H., *J. Am. Ceram. Soc.*, 1981, **64**, 617.
- 10. Park, M. B., Kim, C. D. and Cho, N. H., *Mater. Sci. Eng. B*, 2003, **99**, 15.
- 11. Ueoka, H., *Ferroelectrics*, 1974, **7**, 351.
- 12. Ting, C.-J., Peng, C.-J., Lu, H.-Y. and Wu, S.-T., *J. Am. Ceram. Soc.*, 1990, **73**, 329.
- 13. Rase, D. E. and Roy, R., *J. Am. Ceram. Soc.*, 1955, **38**, 389.
- 14. Roseman, R. D. and Mukharjee, N., *J. Electroceram.*, 2003, **10**, 117.
- 15. Ueoka, H. and Yodogana, M., *IEEE Trans. Manuf. Technol. MFT-3*, 1974, 77.
- 16. Cheng, H.-F., *J. Appl. Phys.*, 1989, **66**, 1382.
- 17. Abicht, H. P., Langhammer, H. T. and Felgner, K. H., *J. Mater. Sci.*, 1991, **26**, 2337.
- 18. Jonker, G. H., *Mater. Res. Bull.*, 1967, **2**, 401.
- 19. Ihring, H. and Puschert, W., *J. Appl. Phys.*, 1977, **48**, 3081.
- 20. Matsuoka, T., Matsuo, Y., Sasaki, H. and Hayakawa, S., *J. Am. Ceram. Soc.*, 1971, **55**, 108.
- 21. IIIingsworth, J., Al-Allak, H. M., Brinkman, A. W. and Woods, J., *J. Appl. Phys.*, 1990, **67**, 2088.
- 22. Allak, H. M., Brinkman, A. W., Russell, G. J. and Woods, J., *J. Appl. Phys.*, 1988, **63**, 4530.
- 23. Garthsen, P., Groth, R. and Hardtl, K. H., *Phys. Status Solidi*, 1965, **11**, 303.
- 24. Brzozowski, E. and Castro, M. S., *J. Eur. Ceram. Soc.*, 2004, **24**, 2499.
- 25. Ihring, H. and Hennings, D., *Phys. Rev. B*, 1978, **17**, 4593.
- 26. Gerthsen, P. and Hoffmann, B., *Solid State Electron.*, 1973, **16**, 617.